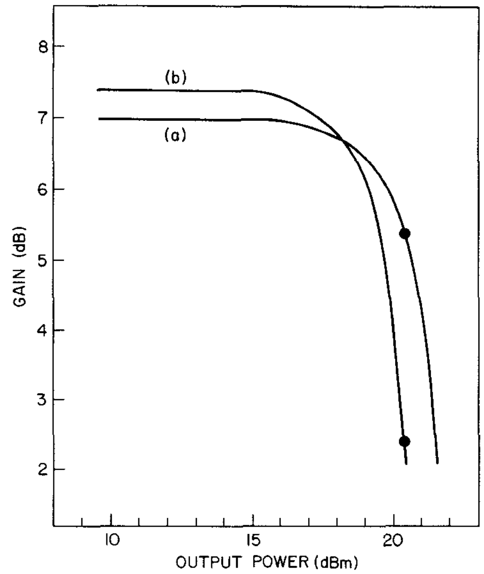

Fig. 10. Gain compression as a function of output power measured at 12 GHz. (a) Response of the 8- to 12-GHz amplifier optimized for large-signal performance. (b) Response of the same amplifier, but with its output circuit retuned externally for maximum *small-signal* gain.

output port retuned for maximum *small-signal* gain. The retuning of the output was simulated with the aid of an external tuning arrangement [5]. This experiment quantitatively illustrates the reduction in large-signal gain which would result if achieving the nominal output power level had been attempted with only a small-signal design.

IV. Conclusions

A systematic procedure has been described for designing GaAs FET power amplifiers for optimum large-signal gain performance. The technique has been applied, specifically, to broadband quasi-class-A circuits. The principal merits of the method include remarkable simplicity and numerical efficiency of the overall design procedure, reliability in predicting large-signal amplifier performance, and the need to acquire experimental large-signal data only at one particular frequency. The viability of this approach has been verified through various comparisons between measured and predicted results for three individual devices exhibiting considerably differing geometries and electrical characteristics.

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Power Combining Ladder Network with Many Active Devices

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***Abstract*—This paper presents a thoretical treatment of a line array of van der Pol oscillators mutually coupled by inductances and connected to a load (i.e., a multiple-device ladder oscillator) aiming to investigate its power-combining capability. A mode analysis approach is used, and it is shown that this system can provide output power just equal to the sum of the available powers from all active devices when it operates at the first mode. In the case where the optimum load is connected at an end of the ladder structure, some stable modes other than the first mode exist, but no stable simultaneous multimodes are found. A method for suppressing undesired modes is discussed. A distributed-line coupled ladder structure is also treated to give a thoretical basis for building a microwave multiple-device ladder oscillator.**

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I. Introduction

RECENTLY, various methods for combining the output powers of several active devices have been presented [1], [2]. They can be classified conveniently into two categories: those which combine outputs of several discrete oscillators coupled together and those which construct a multiple-device oscillator. In the former, choice of the combining (or coupling) network is of primary importance and both tree and chain structures are typical [3]–[5]. Single-step combining has been proposed recently [6]. As for the latter category, the general principle of building a multiple-device oscillator is such that diodes are effectively coupled with a common single cavity [7]–[10], with an exception which connects devices to a load at the center through coaxial lines [11].

This paper treats a ladder network loaded with active devices and connected to a resistive load as shown in Fig. 1. While the network apparently consists of inductively coupled oscillator cells, it belongs essentially to the latter category because each active element couples with a normal mode of the ladder structure. In Sections II and III, after deriving fundamental equations describing the behavior of each normal mode, we discuss of power-combining capability and stability for each mode, together with examination of simultaneous multimode oscillations. Section IV is devoted to description of a method for suppression of undesired modes. Further, in Section V, we treat a distributed-line coupled ladder structure with the object being to offer a theoretical basis for building a microwave multiple-device ladder oscillator.

II. Fundamental Equation

A line array of van der Pol oscillators mutually coupled by inductances or capacitances, was analyzed by Endo and Mori who studied its behavior and investigated the possibility of simultaneous multimode oscillations [12]. Here we treat the same structure, but with a load of conductance g_L connected as shown in Fig. 1, with the object being to investigate its power combining capability.

Supposing that g_L is connected at the lth section, we can write the circuit equations as

$$L_t \frac{di_k}{dt} = v_k - v_{k+1}$$
$$i_{k-1} - i_k = C\frac{dv_k}{dt} + \frac{1}{L}\int v_k dt + (g + g_L\delta_{kl})v_k + i_{ak}, \qquad k = 1, 2, \cdots, N \quad (1)$$

where δ_{kl} is the Kronecker delta and, for the volt-ampere characteristic of the active element, a simple cubic nonlinearity is assumed

$$i_{ak} = -g_1 v_k + \frac{4}{3}\theta v_k^3. \quad (2)$$

Combination of (1) and (2) gives

$$\frac{d^2v_k}{dt^2} + \frac{1}{C}\left(\frac{1}{L} + \frac{2}{L_t}\right)v_k - \frac{1}{CL_t}(v_{k-1} + v_{k+1}) = \frac{1}{C}(g_1 - g - g_L\delta_{kl} - 4\theta v_k^2)\frac{dv_k}{dt}$$

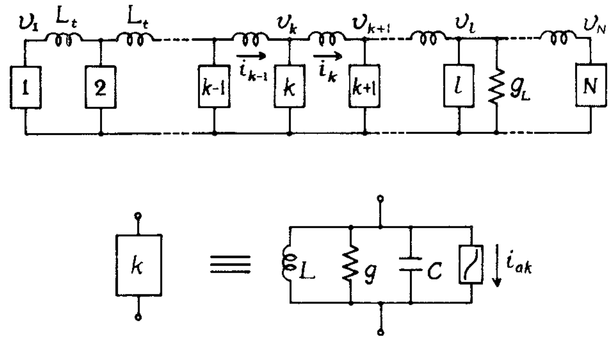

Fig. 1. A power combining multiple-device ladder network.

which can be rewritten as

$$\frac{d^2v_k}{d\tau^2} + (1+2\beta)v_k - \beta(v_{k-1} + v_{k+1}) = \mu(g_0 - g_L\delta_{kl})\frac{dv_k}{d\tau} - \frac{4}{3}\mu\theta\frac{dv_k^3}{d\tau} \quad (3a)$$

where

$$\tau = \omega_0 t \qquad \omega_0^2 = 1/LC$$
$$\beta = L/L_t \qquad \mu = 1/\omega_0 C$$
$$g_0 = g_1 - g. \quad (3b)$$

The boundary condition is given by $i_0 = i_{N+1} = 0$ which, by use of (1a), leads to

$$v_0 = v_1$$
$$v_N = v_{N+1}. \quad (4)$$

Then (3a) can be expressed by a vector differential equation

$$\frac{d^2\boldsymbol{v}}{d\tau^2} + \boldsymbol{Bv} = \mu\left[(g_0\boldsymbol{E} - g_L\boldsymbol{D}_l)\frac{d\boldsymbol{v}}{d\tau} - \frac{4}{3}\theta\frac{d\boldsymbol{V}}{d\tau}\right] \quad (5a)$$

with

$$\boldsymbol{v} = [v_1, v_2, \cdots, v_N]^t$$
$$\boldsymbol{V} = [v_1^3, v_2^3, \cdots, v_N^3]^t \quad (5b)$$

and

$$\boldsymbol{B} = \begin{bmatrix} 1+\beta & -\beta & \cdots & & 0 \\ -\beta & 1+2\beta & \cdots & & \\ & \cdots & \cdots & 1+2\beta & -\beta \\ 0 & & \cdots & -\beta & 1+\beta \end{bmatrix} \quad (5c)$$

where $\boldsymbol{E}$ is the unit matrix and $\boldsymbol{D}_l$ the matrix in which only the (l, l) element has nonzero value of unity and all the others are zero.

Let us now introduce the normal modes of the unperturbed system which is described by $d^2\boldsymbol{v}/d\tau^2 + \boldsymbol{Bv} = 0$, and transform (5a) into the differential equation describing the oscillation at each mode. The normal modes are represented by the eigenvectors of the matrix $\boldsymbol{B}$, which

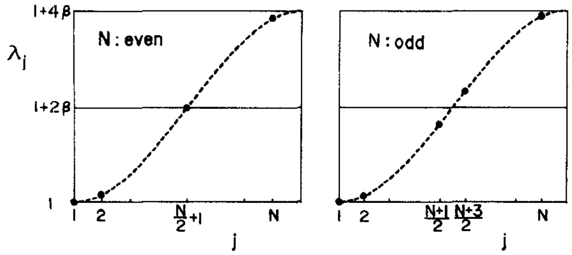

Fig. 2. Distribution of normal mode frequencies $\sqrt{\lambda_j}\ \omega_0$.

easily be obtained as

$$p_j=[p_{1j}p_{2j}\cdots p_{Nj}]^t, \qquad j=1,2,\cdots,N \tag{6a}$$

with

$$p_{kj}=\begin{cases}\sqrt{1/N}, & \text{for } j=1\\ \sqrt{2/N}\cos[(2k-1)(j-1)/2N]\pi, & \text{for } j\geqslant 2\end{cases} \tag{6b}$$

where $k=1,2,\cdots,N$ and $p_i\,p_j^t=\delta_{ij}$ is imposed. The corresponding eigenvalues of B are given by

$$\lambda_j=1+4\beta\sin^2\frac{j-1}{2N}\pi, \qquad j=1,\cdots,N \tag{6c}$$

and $\sqrt{\lambda_j}\ \omega_0$ indicates the frequency of the jth mode oscillation of the unperturbed system (see Fig. 2).

Let us expand v as

$$v=\sum_{j=1}^{N}p_jx_j \tag{7}$$

where x_j's are called the mode variables and are assumed to have the form

$$x_j=A_j\cos(\lambda_j^{1/2}\tau+\psi_j) \tag{8}$$

and apply the equivalent linearization technique due to Kryloff and Bogoliuboff [13] to the term in (5a) under the assumption of no resonant interaction between modes. Further, if we assume that the mode frequencies are separated sufficiently, the terms including $dx_i/d\tau$ will give only negligible effect on the jth mode oscillation as long as $i\neq j$. Thus, we obtain

$$\frac{d^2x_j}{d\tau^2}+\lambda_jx_j=\mu\Big(\alpha_j-\theta_{jj}A_j^2-\sum_{i(\neq j)}\theta_{ji}A_i^2\Big)\frac{dx_j}{d\tau} \tag{9a}$$

where

$$\alpha_j\equiv g_0-g_Lp_{lj}^2$$
$$\theta_{jj}\equiv\theta\sum_{k=1}^{N}p_{kj}^4$$
$$\theta_{ji}\equiv 2\theta\sum_{k=1}^{N}p_{kj}^2p_{ki}^2, \qquad j,i=1,2,\cdots,N. \tag{9b}$$

After Ramb's theory of laser oscillation [14], we may call α_j a gain parameter and θ_{jj} and θ_{ji} self- and mutual-saturation parameters, respectively. Substituting (6b) into (9b), we can determine these parameter values as

$$\alpha_1=g_0-g_L/N$$
$$\alpha_j=g_0-\frac{2g_L}{N}\cos^2\frac{(2l-1)(j-1)}{2N}\pi, \qquad j\geqslant 2 \tag{10a}$$

and

$$\theta_{ji}=\begin{cases}\theta/N, & \text{for } j=i=1,\ \ 1+N/2\\ (3/2)\theta/N, & \text{for } j=i\neq 1,\ \ 1+N/2\\ \theta/N, & \text{for } j+i=N+2\\ 2\theta/N, & \text{for other } j,i.\end{cases} \tag{10b}$$

Using the averaging method under the assumption that A_j and ψ_j in (8) are slowly varying functions of time, we finally obtain the reduced differential equations of (9a) as

$$\frac{dA_j^2}{d\tau}=\mu\Big(\alpha_j-\theta_{jj}A_j^2-\sum_{i\neq j}\theta_{ji}A_i^2\Big)A_j^2$$
$$\frac{d\psi_j}{d\tau}=0. \tag{11}$$

III. Steady State of Optimum Operation

A. Maximum Output Power

Consider a steady state in which only one mode, say the jth mode, is excited. Putting $A_i^2=0$ for all i except j and $dA_j^2/d\tau=0$ in (11), the amplitude of steady oscillation at this mode is given as

$$A_j^2=\frac{\alpha_j}{\theta_{jj}}=A_{j0}^2. \tag{12}$$

When the circuit is connected with a load at the lth section and operates at the jth mode, the voltage of the kth section, $V_k(l,j)$, and the output power $P(l,j)$ are then given, using (7) and (12), as

$$V_k^2(l,j)=p_{kj}^2A_{j0}^2=p_{kj}^2\alpha_j/\theta_{jj} \tag{13}$$

and

$$P(l,j)=\frac{1}{2}g_LV_l^2(l,j)=\frac{1}{2}g_Lp_{lj}^2\alpha_j/\theta_{jj}. \tag{14}$$

As we can see in (6b) and (10) the first and the $(1+N/2)$th modes are of particular interest. So, it will be appropriate to proceed in the following order.

1) $j=1$: This mode has a flat pattern given by $p_{k1}=\sqrt{1/N}$, for all k, and yields the output power

$$P(l,1)=\frac{g_L}{2\theta}\Big(g_0-\frac{g_L}{N}\Big).$$

This is maximized by choosing g_L as

$$g_{L,\mathrm{opt}}(l,1)=(N/2)g_0 \tag{15a}$$

and the maximum value is given by

$$P_{\max}(l,1)=Ng_0^2/(8\theta). \tag{15b}$$

From (15b) we can state that, under first mode operation, perfect power combining is possible regardless of the loading position, because the available power of an active element with the nonlinearity of (2) is $g_0^2/(8\theta)$.[1] In this optimum state the mode amplitude and the oscillation amplitude of each section are given, respectively, by

$$A_{10}^2 = Ng_0/(2\theta) \tag{15c}$$

$$V_k^2(l,1) = g_0/(2\theta), \qquad \text{for all } k. \tag{15d}$$

2) $j=1+N/2$ (N:even): This mode is characterized by $p_{2n,1+N/2} = p_{2n+1,1+N/2} = (-1)^n\sqrt{1/N}$ which gives $p_{k,1+N/2}^2 = P_{k,1}^2$ for all k and leads to the same expression for the output power as in the 1st mode. Accordingly, the operation at this mode also enables perfect power addition and all expressions in (15) also hold for this mode, if we replace the mode number 1 by $1+N/2$.

3) $j\neq 1, 1+N/2$: For all these modes, (14) becomes

$$P(l,j) = \frac{g_L^*}{3\theta}\left(g_0 - \frac{g_L^*}{N}\right)$$

with

$$g_L^* = 2g_L\cos^2\frac{(2l-1)(j-1)}{2N}\pi.$$

The optimum load conductance and the maximum output power are then given as

$$g_{L,\text{opt}}(l,j) = \frac{Ng_0}{4\cos^2[(2l-1)(j-1)/2N]\pi} \tag{16a}$$

$$P_{\max}(l,j) = \frac{Ng_0}{12\theta} = \frac{2}{3}P_{\max}(l,1). \tag{16b}$$

Note that all the modes other than $j=1$ and $j=1+N/2$ cannot provide output power exceeding 2/3 of the available power of the system. The reason why these modes cannot succeed in perfect combining is that all V_k^2's cannot take the value $g_0/(2\theta)$ as in (15d).

B. Stability of Single Modes

Next, we must determine if the power combining mode $j=1$ or $j=1+N/2$ is actually stable and if there exist any other stable modes. In order for the jth mode oscillation to be stable, small variations in A_i^2's around their stationary values must decay in time, and this is the case if all the eigenvalues of a matrix composed of

$$J_{ik} \equiv \left[\frac{\partial}{\partial A_k^2}\left(\frac{dA_i^2}{dt}\right)\right]_{A_j^2 = A_{j0}^2,\, A_{i(\neq j)}^2 = 0}$$

have negative real parts. The matrix elements are written, using (11), as

$$J_{ik} = \begin{cases} -\mu\alpha_j, & \text{for } i=k=j \\ \mu(\alpha_i - \theta_{ij}\alpha_j/\theta_{jj}), & \text{for } i=k\neq j \\ -\mu\theta_{ik}\alpha_j/\theta_{jj}, & \text{for } i\neq k, \quad i=j \\ 0, & \text{for } i\neq k, \quad i\neq j \end{cases} \tag{17}$$

[1] The electronic output power of an active element characterized by (2) is given by $P_{el} = (1/2)(g_0 - \theta V^2)V^2$, which takes the maximum value $g_0^2/(8\theta)$ for $V^2 = g_0/(2\theta)$.

from which the eigenvalues prove to be equal to J_{ii}, $i=1,2,\cdots,N$. The stability condition for the jth mode is then expressed as

$$\alpha_i/\theta_{ij} < \alpha_j/\theta_{jj}, \qquad \text{for all } i(\neq j). \tag{18}$$

This condition physically means that the effective gain for the ith mode $\alpha_i - \theta_{ij}A_{j0}^2$ is negative due to the presence of the jth mode, which makes the ith mode oscillation impossible.

Next we examine stability of each mode for two typical cases, that in which the optimum load is at the end section and that in which the load is at the central section.

1) Case of End Loading: When a load of optimum conductance $(1/2)Ng_0$ as given in (15a) is connected to the end section ($l=N$), gain parameters can be expressed as

$$\alpha_1 = (1/2)g_0$$

$$\alpha_j = g_0\sin^2\frac{j-1}{2N}\pi, \qquad j\geq 2 \tag{19}$$

which yields, together with (10b),

$$\frac{\alpha_i}{\theta_{ij}} = \frac{Ng_0}{2\theta}\times\begin{cases} 1, & \text{for } i=j=1,\ 1+N/2 \\ (4/3)\sin^2[(i-1)\pi/2N], & \text{for } i=j\neq 1,\ 1+N/2 \\ 2\sin^2[(i-1)\pi/2N], & \text{for } i+j=N+2 \\ 1/2, & \text{for } i=1, j\neq 1 \\ \sin^2[(i-1)\pi/2N], & \text{for other } i,j. \end{cases} \tag{20}$$

By using (20) in the stability condition (18), we find that the stable modes include the power-combining modes $i=1$, $1+N/2$ and some higher modes specified by $j\geq(2/3)N+1$.

2) Case of Central Loading: First, for odd N, l is taken $(N+1)/2$ and (10a) gives

$$\alpha_1 = g_0/2$$

$$\alpha_j = g_0\sin^2\frac{j-1}{2}\pi = \begin{cases} g_0, & \text{for even } j \\ 0, & \text{for odd } j. \end{cases} \tag{21}$$

Then we have

$$\frac{\alpha_i}{\theta_{ij}} = \frac{Ng_0}{2\theta}\times\begin{cases} 1/2, & \text{for } i=1, \quad j\neq 1 \\ 4/3, & \text{for } i=j=2,4,\cdots,N-1 \\ 2, & \text{for } i+j=N+2, i=2,4,\cdots,N-1 \\ 0, & \text{for } i=3,5\cdots,N \\ 1, & \text{for other } i,\ j \end{cases} \tag{22}$$

and, by application of (18), it turns out that the stable modes are those with even mode number, all of which have voltage standing wave nodes at the central section and give no output power.

Next, for even N, supposing that the load is connected to the middle point of the $(N/2)$th and the $(N/2+1)$th section,[2] we have

$$\alpha_1 = g_0 - g_L\left\{\frac{1}{2}(p_{N/2,1} + p_{N/2+1,1})\right\}^2 = \frac{1}{2}g_0$$

$$\alpha_j = g_0 - g_L\left\{\frac{1}{2}(p_{N/2,j} + p_{N/2+1,j})\right\}^2$$

$$= \begin{cases} g_0, & \text{for } j = 2,4,\cdots,N \\ g_0 \sin^2 \dfrac{j-1}{2N}\pi, & \text{for } j = 3,5,\cdots,N-1 \end{cases} \tag{23}$$

which, together with (10b), gives

$$\frac{\alpha_i}{\theta_{ij}} = \frac{Ng_0}{2\theta} \times \begin{cases} 1/2, & \text{for } i = 1, 1+N/2 \\ & \text{(when } N/2 \text{ is even)}; j \neq i \\ 4/3, & \text{for } i = j = 2,4,\cdots,N \\ \dfrac{4}{3}\sin^2\dfrac{(j-1)\pi}{2N}, & \text{for } i = j = 3,5,\cdots,N-1 \\ 2, & \text{for } i+j = N+2, \\ & \quad i = 2,4,\cdots,N \\ 2\sin^2\dfrac{(i-1)\pi}{2N}, & \text{for } i+j = N+2, \\ & \quad i = 3,5,\cdots,N-1 \\ 1, & \text{for other } i, j. \end{cases} \tag{24}$$

Inspection of stability indicates that the stable modes are those with odd mode number satisfying $j > (2/3)N + 1$ and, when $N/2$ is odd, the $(1+N/2)$th mode is also stable.

The conclusion is that central loading cannot ensure the stable power-combining mode and some scheme encouraging the wanted mode while suppressing the unwanted stable modes will be needed.

C. Possibility of Simultaneous Multimode Oscillations

In general, systems with many degrees of freedom have the possibility of simultaneous multimode oscillations and it is known that some stable double mode oscillations exist in a ladder oscillator with no load [12]. Here we give a brief discussion of this problem concerning the loaded ladder oscillator.

Taking a procedure similar to that for the single-mode case, we can find the necessary and sufficient condition for the stable simultaneous double-mode oscillation as

$$G_{ii} > G_{ji}$$
$$G_{jj} > G_{ij} \tag{25a}$$

and

$$G_{ki}(G_{jj} - G_{ij}) + G_{kj}(G_{ii} - G_{ji}) > G_{ii}G_{jj} - G_{ij}G_{ji}, \quad k \neq i, j \tag{25b}$$

where

$$G_{ij} = \theta_{ij}/\alpha_i. \tag{26}$$

[2] In this connection to the load, the maximum output power at the first mode is still given by (15b).

TABLE I
LIST OF STABLE MODES

	end-loading case	central-loading case	
		N : odd	N : even
stable single mode	$j=1,\ N/2+1,$ $j \geq 2N/3+1$	$j=2,4,\cdots$ $\cdots, N-1$	$j > 2N/3+1$ for all even N $j = N/2+1$ for odd $N/2$
stable double mode	none	none	$\{\frac{N}{2}+1-(2\nu+2),\ \frac{N}{2}+1+(2\nu+2)\}$ $\nu = 0,1,2,\ldots,(N-2)/4-1$ for odd $N/2$ $(N \geq 6)$ $\{\frac{N}{2}+1-(2\nu+1),\ \frac{N}{2}+1+(2\nu+1)\}$ $\nu = 0,1,2,\ldots,N/4-1$ for even $N/2$ $(N \geq 4)$

Results of applying (25a) and (25b) to our problem can be summarized as follows.

1) For the case of end-loading, we can find some pairs of modes relating to each other under (25a) as

$$\left\{\frac{N}{2}+1-(\nu+1),\ \frac{N}{2}+1+(\nu+1)\right\}, \quad \text{with } \nu = 0,1,\cdots, \text{ for even } N(N \geq 16)$$

$$\left\{\frac{N}{2}+1-\left(\nu+\frac{1}{2}\right),\ \frac{N}{2}+1+\left(\nu+\frac{1}{2}\right)\right\}, \quad \text{with } \nu = 0,1,\cdots, \text{ for odd } N(N \geq 9). \tag{27}$$

But these mode-pairs cannot satisfy the second condition (25b), so stable nonresonant double-modes are impossible for the endloading case.

2) Next, for the case of central-loading, we obtain different results according as N is odd or even. For odd N, there can be no stable double-modes because none of mode pairs satisfies (25a). For even $N(\geq 4)$, on the other hand, some stable double modes can be found as listed in Table I.

Further, it is rather easy to conclude that there exist no stable multimode oscillations with higher multiplicity for either the end-loading or central-loading case, because we cannot find any mode triads satisfying

$$\begin{vmatrix} G_{ii} & G_{ij} & G_{ik} \\ G_{ji} & G_{jj} & G_{jk} \\ G_{ki} & G_{kj} & G_{kk} \end{vmatrix} > 0$$

which is merely one of the necessary conditions for stable triple-mode oscillation. The results of Sections III-B and III-C are summarized in Table I.

IV. Suppression of Undesired Modes

According to the analysis of the preceding section, stable operation at the power combining mode—say, at the first mode—requires suppression of unwanted stable modes and, for the central loading case, also turning the

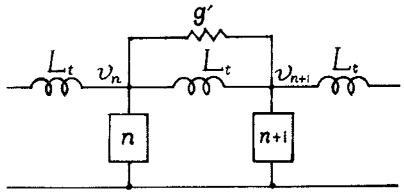

Fig. 3. Introduction of a loss conductance for mode suppression.

wanted mode into a stable one. To meet these requirements, let us consider a system furnished with a conductance g' between the nth and the $(n+1)$th section as shown in Fig. 3. In this case, the circuit equation (1b) should have added to it:

$$g'(\delta_{k,n}-\delta_{k,n+1})(v_n-v_{n+1})$$

on the right side, which results in addition of

$$-\mu g' \boldsymbol{D} dv/dt$$

on the right side of (5a), $\boldsymbol{D}$ being a matrix with $D_{n,n}=D_{n+1,n+1}=1$, $D_{n,n+1}=D_{n+1,n}=-1$ and all other elements being zero. In terms of the mode variables, this gives rise to an additional term

$$-\mu g'(p_{nj}-p_{n+1,j})^2\frac{dx_j}{dt}$$

on the right side of (9a), and we arrive at a modified reduced equation

$$\frac{dA_j^2}{d\tau}=\mu\Big(\alpha_j'-\theta_{jj}A_j^2-\sum_{i(\neq j)}\theta_{ji}A_i^2\Big)A_j^2 \tag{28a}$$

where

$$\alpha_j'=g_0-g_Lp_{lj}^2-g'(p_{n,j}-p_{n+1,j})^2. \tag{28b}$$

For the first mode, we have $\alpha_i'=\alpha_i$ because $p_{n,1}-p_{n+1,1}=0$ irrespective of n and find no effect due to introduction of g', so we can still use various relations given in (12)–(15). Substituting (6b) into (28b), we get

$$\alpha_j'=\begin{cases}\alpha_1, & \text{for } j=1\\ \alpha_j-\dfrac{8g'}{N}\sin^2\dfrac{n(j-1)\pi}{N}\sin^2\dfrac{(j-1)\pi}{2N}, & \text{for } j\geqslant 2.\end{cases} \tag{29}$$

As the reduced equation (28a) is of the same form as the original one, that is (11), the stability condition (18) still holds but only if α_j is replaced by α_j'.

A. Suppression of Undesired Single Modes

1) Case of End Loading: While the first mode gain parameter is not diminished due to introduction of g', all the other modes innevitably undergo attenuation as we can see in (29), so the first mode remains stable. Then, what is needed is to determine the values of n and g' which make all the other modes unstable. Apparently, a sufficient condition for this purpose is given by

$$\alpha_j'<(\theta_{jj}/\theta_{1j})\alpha_1, \qquad \text{for } j\neq 1 \tag{30}$$

where, from (19) and (29),

$$\alpha_j'=\Big[g_0-\frac{8g'}{N}\sin^2\frac{n(j-1)\pi}{N}\Big]\sin^2\frac{(j-1)\pi}{2N}, \qquad j\neq 1$$

and, from (10b) and (19),

$$\frac{\theta_{jj}}{\theta_{ij}}\alpha_1=\begin{cases}(3/8)g_0, & \text{for } j\neq 1, 1+N/2\\ (1/4)g_0, & \text{for } j=1+N/2.\end{cases}$$

As is easily noted, in order for (30) to be satisfied, it is necessary and sufficient that (30) holds for the modes $j=N$ and $j=1+N/2$. Thus, (30) reduces to

$$g'\sin^2\frac{n\pi}{N}>\frac{N}{8}\Big(1-\frac{3}{8}\sec^2\frac{\pi}{2N}\Big)g_0, \qquad \text{both for even and odd } N \tag{31a}$$

and

$$g'\sin^2\frac{n\pi}{2}>\frac{N}{16}g_0, \qquad \text{for even } N, \tag{31b}$$

which determine the optimum value of n; n_{opt}, as

$$n_{\text{opt}}=\begin{cases}N/2, & \text{for odd } N/2\\ N/2\pm 1, & \text{for even } N/2\\ (N\pm 1)/2, & \text{for odd } N.\end{cases} \tag{32}$$

With n replaced by n_{opt}, the optimum loss conductance is then determined so as to satisfy (31). Note that, for large N, the g' value must be increased proportionately with N.

2) Case of Central Loading: In the original system ($g'=0$), we had $\alpha_i/\theta_{i1}\leqslant\alpha_1/\theta_{11}$, $i\neq 1$ instead of (18) with $j=1$ and the stability of the first mode could not be ensured. But, upon introducing g', gain parameters of all other modes can be lowered through proper choice of n, whereas that of the first mode remains unchanged. Thus the power-combining mode can be altered to a stable one.

In order to suppress all the undesired modes, we can proceed in the same manner as in the previous case. In the present case, the condition for mode suppression (30) leads to

$$g'\sin^2\frac{n\pi}{N}\sin^2\frac{\pi}{2N}>\frac{5}{64}Ng_0, \qquad \text{both for even and odd } N \tag{33a}$$

and

$$g'\sin^2\frac{n\pi}{2}>\frac{3}{16}Ng_0, \qquad \text{for odd } N/2$$
$$g'\sin^2\frac{n\pi}{2}>\frac{1}{16}Ng_0, \qquad \text{for even } N/2 \tag{33b}$$

which again produce the same expression (32) for n_{opt}. In this case, as N is increased, the g' value must rise in proportion to $N/\sin^2(\pi/2N)$, a faster rate than in the end-loading case.

B. Suppression of Double Modes

It was shown in Section III-C that the end-loading case and the central-loading case with odd N have no stable double mode, and our concern here is whether or not all

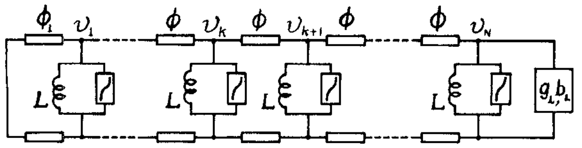

Fig. 4. A distributed-line coupled ladder structure.

the mode pairs remain unstable after introduction of g'. Results of analysis are summarized below.

1) End-Loading Case: It is easy to see that the mode pairs (27) still satisfy the weak-coupling relation (25a). A sufficient condition for these mode pairs to remain unstable is to choose g' as

$$g'\cos^2\frac{N+1}{4N}\pi > \frac{3}{64}Ng_0, \qquad \text{for odd } N(N\geqslant 9). \quad (34)$$

For even N, suppression of the mode pairs (27) can be achieved through introduction of additional loss conductance g'' at the first section ($n=1$). A sufficient condition for this purpose is given by

$$g''\cos^2\frac{\pi}{N} > \frac{3}{64}Ng_0, \qquad \text{for even } N(N\geqslant 16). \quad (35)$$

2) Central-Loading Case: The situation in which no mode pairs with weak-coupling conditions exist remains unchanged irrespective of the g' value when N is odd.

V. Distributed-Line Coupled Ladder Oscillator

In this section, we treat the system shown in Fig. 4 as a microwave version of the ladder oscillator discussed in the preceding sections. In the figure, ϕ_1 and ϕ denote the electrical length of the coupling lines, and g_L and b_L are the load conductance and susceptance seen at the Nth section.

In the steady state, in which all the sections oscillate at the same amplitude and phase as in the first mode operation of the system of Fig. 1, the middle point of each coupling line must be the position of a voltage standing wave maximum. So, we set the following relations between circuit parameters:

$$\begin{aligned} -Y_0\cot\phi_1 - \frac{1}{\omega L} + Y_0\tan\frac{\phi}{2} &= 0 \\ Y_0\tan\frac{\phi}{2} - \frac{1}{\omega L} + Y_0\tan\frac{\phi}{2} &= 0 \qquad (36) \\ Y_0\tan\frac{\phi}{2} - \frac{1}{\omega L} + b_L Y_0 &= 0, \qquad \text{for } N\geqslant 3 \end{aligned}$$

which are reduced to the simpler form

$$-\cot\phi_1 = b_L, \qquad \text{for} N\geqslant 2 \quad (37a)$$

and

$$-\cot\phi_1 = \tan\phi/2, \qquad \text{for } N\geqslant 3. \quad (37b)$$

Then, for ϕ smaller than π, by putting

$$\omega C \equiv Y_0(-\cot\phi_1 + \tan\phi/2) \quad (38a)$$

$$-1/\omega L_t \equiv -Y_0\operatorname{cosec}\phi \quad (38b)$$

the present system can be transformed to a lumped-constant system of the same structure as discussed in the preceding sections, namely that of Fig. 1. However, several comments must be added on this transformation.

First, C and L_t defined by (38) are necessarily frequency dependent, and this means that the values of these parameters vary from mode to mode. This may introduce difficulty in that the value of β, which appears in the elements of the matrix $\boldsymbol{B}$ in the discussion of Section II, also varys with modes. Fortunately, we have the definite form of $\boldsymbol{B}$ as given in (5c) regardless of ununiqueness of β. As a consequence, normal modes, as the eigenvectors of $\boldsymbol{B}$, can be determined uniquely and still expressed by (6b), which in turn ensures that α_j and θ_{ji} are given uniquely by (9b). And after all, equation (18) remains effective as the condition for stability. As for the eigenvalues of $\boldsymbol{B}$, the expression for λ_j can be given definitely by (6c) if β is understood to have the jth mode value. A procedure for obtaining the jth mode frequency will be described below in Section V-A.

Second, for $N\geqslant 3$, it is generally not the case that (37) can be satisfied by all the higher modes as well as by the first mode. So, analysis based on the assumption that (37) holds for every mode cannot give correct results for the modes $j\geqslant 2$. However, rigorous description of the first mode operation and its stability is safely possible (see Section V-A).

Third, the assumption $0<\phi<\pi$ made when defining C and L_t may fail for some higher modes. However, all the discussion of the behavior of each mode under the assumption $0<\phi<\pi$ can safely be applied for the modes for which ϕ exceeds π (see Appendix).

With the above limitations in mind, we can use Fig. 1 as an equivalent circuit for our distributed-line coupled ladder oscillator.

A. Operation at First Mode and Its Stability

The first mode operation can be achieved if we design the circuit of our oscillator using (37a) and (37b) and, according to the analysis in Section II, it gives perfectly combined power output. The oscillation frequencies of each mode are given, using (6c) and (3b), by

$$\omega_j^2 = \left(\frac{1}{L} + \frac{4}{L_t}\sin^2\frac{j-1}{2N}\right)\frac{1}{C}, \qquad \text{for } j=1,2,\cdots,N$$

or, returning to the original parameters ϕ and ϕ_1, by

$$Y_0\left(-\cot\phi_1 + \tan\frac{\phi}{2} - 4\sin^2\frac{j-1}{2N}\pi\cdot\operatorname{cosec}\phi\right) = \frac{1}{\omega L},$$

$$\text{for } j=1,2,\cdots,N. \quad (39)$$

Note that (39) reduces to the first equation of (36) if we set $j=1$.

Next we must determine whether the first mode oscillation is again stable in the present system designed to meet (37). As stated before, the relation (37) cannot hold for other modes; accordingly there occurs some modification in p_{ki} for $i\neq 1$. But, by virtue of the independency of p_{k1}

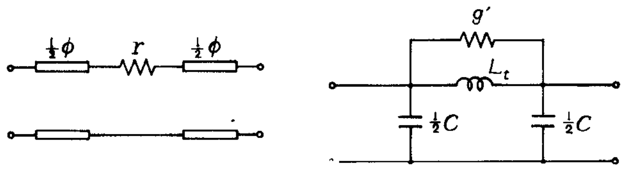

Fig. 5. Insertion of a loss into a coupling line (a) and its equivalent circuit (b).

on k

$$\theta_{i1}=2\theta\sum_{k}^{N} p_{ki}^2 p_{k1}^2=\frac{2\theta}{N}\sum p_{ki}^2=\frac{2\theta}{N}, \qquad \text{for } i\neq 1$$

that is, θ_{i1} undergoes no alteration regardless of the p_{ki} value. Then, since $\alpha_i=g_0\{1-(N/2)p_{ni}^2\}<g_0$ irrespective of the p_{Ni} value, we obtain

$$\alpha_i-\theta_{i1}(\alpha_1/\theta_{11})=\alpha_i-g_0<0, \qquad \text{for } i\neq 1$$

and this ensures the stability of the first mode.

B. Suppression of Undesired Modes

Concerning stabilities of all the other modes, that is, of the undesired modes, it is almost impossible to deduce a definite conclusion because deviations from (37) differ with modes. In any case, it is obviously necessary to introduce a device for suppressing undesired modes as long as some of them are possibly stable.

Following the scheme discussed in Section IV, consider a coupling line inserted with a resistance r at the middle point, where only the first mode has no electric current (see Fig. 5(a)). The Y-matrix of the line section is given by

$$Y_{11}=Y_{22}=\frac{-j\cot\phi+rY_0/2}{1-j(rY_0/2)\cot(\phi/2)}Y_0$$
$$\doteqdot\left[-j\cot\phi+\frac{1}{2}rY_0\left(1+\cot\phi\cot\frac{\phi}{2}\right)\right]Y_0$$
$$Y_{12}=Y_{21}=\frac{j\operatorname{cosec}\phi}{1-j(rY_0/2)\cot(\phi/2)}Y_0$$
$$\doteqdot\left[j\operatorname{cosec}\phi-\frac{1}{2}rY_0\operatorname{cosec}^2\frac{\phi}{2}\right]Y_0 \qquad (40)$$

where the approximate expressions hold if $rY_0\cot(\phi/2)\ll 1$. Then putting

$$g'\equiv(1/4)rY_0\operatorname{cosec}^2(\phi/2)\cdot Y_0$$
$$-1/(\omega L_t)\equiv-Y_0\operatorname{cosec}\phi$$
$$\omega(C/2)\equiv Y_0\tan(\phi/2) \qquad (41)$$

we get the equivalent circuit shown in Fig. 5(b) which exactly corresponds to Fig. 3, and can behave as the way described in Section VI.

VI. Conclusion

We have shown that a multiple-device oscillator using a ladder network can provide output power just equal to the sum of the available powers from each active device when it operates at the first (fundamental) mode, and discussed the stability of this mode together with a method for suppressing undesired modes.

In the present paper we used mode-analysis technique to discuss moding problem, but this technique fails to give the phase difference between two adjacent sections, which must exist in consistency with the power flow from each active device to the load. For derivation of the proper phase relations, it is necessary to take another approach, say, steady-state analysis for the system operated at a desired mode. This additional work is being carried out. Regarding the problem of elimination of multimodes in the end-loading ladder oscillator, a useful comment can be added from a practical standpoint: in view of (6c) and Table I, the oscillation frequencies of the stable undesired modes can be well separated from that of the first mode by using a large value for the coupling parameter β; thus, the number of undesired modes to be suppressed is reduced depending on the working range of the active devices and, if fortunate, only the first mode can appear without introducing any loss conductances.

Although the purpose of this paper is to present a theoretical study on a multiple-device ladder network, it seems encouraging to note that its power-combining capability has already been verified by experiments in the microwave region[3] [16], [17]. More detailed and extensive study toward practical use is needed. Also, application of the multiple-device ladder network to construction of a high power amplifier would be an interesting subject for further work.

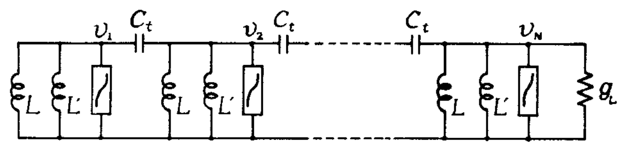

Fig. 6. Another equivalent circuit for a distributed-line coupled ladder structure.

Appendix

Treatment of the Modes for which $\pi<\phi<2\pi$

For these modes, instead of (38a) and (38b), we set

$$-1/\omega L'=Y_0(-\cot\phi_1+\tan\phi/2)$$
$$\omega C_t=-Y_0\operatorname{cosec}\phi \qquad (42)$$

and obtain Fig. 6 as an equivalent circuit of the system. Then, the differential equation of the system becomes

$$\boldsymbol{B}'\frac{d^2\boldsymbol{v}}{d\tau^2}+\boldsymbol{v}=\mu'(g_0\boldsymbol{E}-g_L\boldsymbol{D}_l)\frac{d\boldsymbol{v}}{d\tau}-\frac{4}{3}\mu'\theta\frac{d\boldsymbol{V}}{d\tau} \qquad (43a)$$

[3]The desired-mode operation has been successfully obtained under the optimum circuit condition (36). Under other conditions far from (36), undesired modes can appear, but they are easily suppressed by use of the technique as stated in Section IV. As typical data, we obtained the output power of about 100, 98, and 96 percent of the sum of available powers of each diode (Gunn diode) for quadruple-, sextuple-, and octuple-diode ladder structure, respectively.

with

$$B'=\begin{vmatrix} 1 & -1 & & & 0 \\ -1 & 2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & 2 & -1 \\ 0 & & & -1 & 1 \end{vmatrix} \qquad (43b)$$

and

$$\tau=\omega_0 t, \qquad \omega_0^2=\left(\frac{1}{L}+\frac{1}{L'}\right)\frac{1}{C_t}$$

$$\mu'=1/(\omega_0 C_t). \qquad (43c)$$

The eigenvalues of $\boldsymbol{B}'$ are given by

$$\lambda_j'=4\sin^2\frac{j-1}{2N}\pi, \qquad j=1,2,\cdots,N \qquad (44)$$

and the jth mode frequency of the unperturbed system of (43a) is written as

$$\omega_j^2=(\lambda_j')^{-1}\omega_0^2=\frac{(1/C_t)(1/L+1/L')}{4\sin^2(j-1)\pi/2N}$$

which is reduced to the same expression as (39) by using (42). On the other hand, the eigenvectors of $\boldsymbol{B}'$ and accordingly of $(\boldsymbol{B}')^{-1}$ are the same as those for $\boldsymbol{B}$. Thus, both equivalent circuit representation of Fig. 1 and Fig. 6 gives the same normal modes and the same mode-frequencies.

As before, transforming the variables from v_k's to x_j's by (7) and assuming $x_j=A_j\cos\{(\lambda_j')^{-1/2}\tau+\psi_j\}$, we obtain

$$\frac{d^2x_j}{d\tau^2}+\frac{1}{\lambda_j'}x_j=\frac{\mu'}{\lambda_j'}\left[\alpha_j-\theta_{jj}A_j^2-\sum_{\substack{i=1\\(i\neq j)}}^{N}\theta_{ji}A_i^2\right]x_j \qquad (45)$$

where α_j's and θ_{ij}'s are exactly the same as given in (10). Reduced differential equations of (45) then become

$$\frac{dA_j^2}{d\tau}=\frac{\mu'}{\lambda_j'}\left[\alpha_j-\theta_{jj}A_j^2-\sum_{\substack{i=1\\(i\neq j)}}^{N}\theta_{ji}A_i^2\right]A_j^2,$$

$$\frac{d\psi_i}{dt}=0. \qquad (46)$$

Thus, if we suppose that $0<\phi<\pi$ for modes $1\leqslant j\leqslant J$ and $\pi<\phi<2\pi$ for modes $J+1\leqslant j\leqslant N$, appropriate reduced equations will be (11) and (46) according as $1\leqslant j\leqslant J$ and $J+1\leqslant j\leqslant N$. But we notice that (46) is the same as (11) except for the coefficient of the right side. So the stability condition for the jth mode is still given by (18) and this justifies discussion of the behavior of each mode entirely on the basis of (11).

To sum up, a physically reasonable equivalent circuit for a distributed-line coupled ladder oscillator of Fig. 4 is given by Fig. 1 or by Fig. 6 according as $\phi<\pi$ or $\pi<\phi<2\pi$. Mathematical description of the behavior of each mode, especially of the competitive relation between modes, is possible using either of them.

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